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**Mathematics**  
**Higher level**  
**Paper 3 – calculus**

Tuesday 10 November 2020 (afternoon)

1 hour

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**Instructions to candidates**

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 7]

Use l'Hôpital's rule to find

$$\lim_{x \rightarrow 1} \frac{\cos(x^2 - 1) - 1}{e^{x-1} - x}.$$

2. [Maximum mark: 10]

Consider the series  $\sum_{n=1}^{\infty} \frac{(5-3x)^n}{n}$ .

(a) Show that the series is convergent for  $\frac{4}{3} < x < 2$ . [5]

(b) Find the interval of convergence of the series. [5]

3. [Maximum mark: 18]

The curve  $y = f(x)$  has a gradient function given by

$$\frac{dy}{dx} = x - y.$$

The curve passes through the point (1, 1).

(a) (i) On the same set of axes, sketch and label isoclines for  $\frac{dy}{dx} = -1, 0$  and  $1$ , and clearly indicate the value of each  $y$ -intercept.

(ii) Hence or otherwise, explain why the point (1, 1) is a local minimum. [6]

(b) Find the solution of the differential equation  $\frac{dy}{dx} = x - y$ , which passes through the point (1, 1). Give your answer in the form  $y = f(x)$ . [8]

(c) (i) Explain why the graph of  $y = f(x)$  does not intersect the isocline  $\frac{dy}{dx} = 1$ .

(ii) Sketch the graph of  $y = f(x)$  on the same set of axes as part (a)(i). [4]

4. [Maximum mark: 15]

The function  $f$  is defined by  $f(x) = \ln(1 + x^2)$  where  $-1 < x < 1$ .

(a) (i) Use the Maclaurin series for  $\ln(1 + x)$  to write down the first three non-zero terms of the Maclaurin series for  $f(x)$ .

(ii) Hence find the first three non-zero terms of the Maclaurin series for  $\frac{x}{1+x^2}$ . [6]

(b) Use your answer to part (a)(i) to write down an estimate for  $f(0.4)$ . [1]

The seventh derivative of  $f$  is given by  $f^{(7)}(x) = \frac{1440x(x^6 - 21x^4 + 35x^2 - 7)}{(1+x^2)^7}$ .

(c) (i) Use the Lagrange form of the error term to find an upper bound for the absolute value of the error in calculating  $f(0.4)$ , using the first three non-zero terms of the Maclaurin series for  $f(x)$ .

(ii) With reference to the Lagrange form of the error term, explain whether your answer to part (b) is an overestimate or an underestimate for  $f(0.4)$ . [8]